

Work-Energy

eg.

1. You pull a 45kg bale of hay with 450N of force at 28° to the horizontal over 4.0 m. If the force of friction is 250N determine work done by:

a) you

$$W = Fd \cos \theta = 450 \text{ N} (4.0 \text{ m}) \cos 28^\circ \\ = 1600 \text{ J}$$

a) friction

$$W = Fd = 250 \text{ N} \times 4.0 \text{ m} \\ = -1.0 \times 10^3 \text{ J} \quad \text{Not a Vector}$$

work is negative when F is opposite d
loss of energy

b) net force

$$W = Fd = (450 \cos 28^\circ - 250) \times 4 = 590 \text{ J}$$

or W by applied force - W by friction

c) normal force $W=0$ F perpendicular to d

d) gravity $W=0$

1. A spring has a length of 20.0 cm when not extended. If you hang a 500. g mass on it, it extends to 32.0cm.

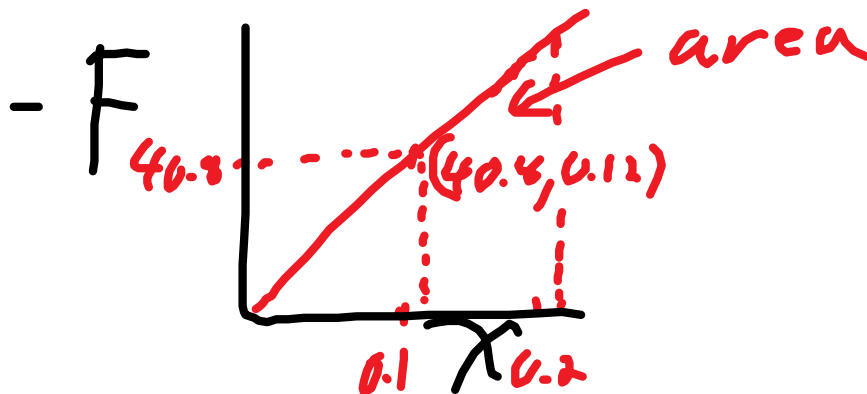
what is the elastic constant, k ?

$$F_{\text{elastic}} = -kx$$

$$k = -F/x = - (0.50\text{kg} \times 9.8\text{N/kg})/(-0.12\text{m})$$

$$k = 40.8\text{N/m}$$

graph F_{elastic} VS x




how much work is done pulling the 500g mass down another 10.0 cm (from length 32.0cm to 42.0cm)

$W =$ area under the F - x graph

(=integral of the F - x function)

$$\int_{x_1}^{x_2} F dx = \int Kx dx = \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2$$

or area of trapezoid = $\frac{1}{2} (b_1 + b_2) h$


$$F_2 = b_2 = \frac{1}{2} (F_2 + F_1) (x_2 - x_1)$$
$$h = x_2 - x_1 = \frac{1}{2} (Kx_2 + Kx_1) (x_2 - x_1)$$

$$= \frac{1}{2} (Kx_2^2 - Kx_2x_1 + Kx_2x_1 - Kx_1^2)$$
$$= \frac{1}{2} Kx_2^2 - \frac{1}{2} Kx_1^2$$

Elastic Energy $E_{\text{elastic}} = \frac{1}{2}Kx^2$ relative to
Energy = 0 when $x = 0$ so our constant
= 0

$$= \frac{1}{2} (40.8 \text{ N/m}) [(0.22 \text{ m})^2 - (0.12 \text{ m})^2]$$

$$= 0.5 \times 40.8 \times (0.22^2 - 0.12^2) = 0.6936$$

0.694 J of energy is required

c) if you let go after c, what is the

acceleration of the 500g mass?

$$\begin{aligned} a &= F_{\text{net}}/m = (F_{\text{elastic}} - F_g)/m \\ &= (40.8\text{N/m} \times 0.22\text{m} - 0.5\text{kg} \times 9.80\text{N/kg})/0.5\text{kg} \\ &= (40.8 \times 0.22 - 0.5 \times 9.8)/0.5 = 8.152 \\ &8.15\text{m/s}^2 \end{aligned}$$

Other forms of energy

kinetic energy, E_k

$$E_k = 1/2 mv^2$$

derived from $W = Fd$ $F = ma$ and $2ad = v_f^2 - v_i^2$

$$= mad = m (v_f^2 - v_i^2)/2 = 1/2 mv_f^2 - 1/2 mv_i^2$$

$W = \text{change in energy}$

work done by the net force = change in kinetic energy

so area under a $F_{\text{net}} - d$ graph is change in kinetic energy

Gravitational Energy, E_g

$E_g = mgh$ if g is constant (near Earth)

derived from $W_g = F_g d = mg\Delta h$

set our reference energy $E_g = 0$ when $h = 0$

eg. You bungee jump from 111m using a

bungee cord that is 10.0m long. If you have a mass of 85.0kg, height 2.0m (centre of mass is 1.0m)

- a) what is your initial gravitational energy?
- b) what is your speed when the cord is about to start stretching?
- c) what elastic constant of the bungee cord will just stop you from hitting the water?
- d) what is your acceleration at the bottom point?
- e) what is the stretch of the bungee cord at equilibrium point (where you stop bouncing)?
- f) how much energy was lost in frictional forces when you are at equilibrium point?